Suppose that $\vec{A} = \vec{A}(x, y, z)$ be a vector function of x, y, z variables. Then we can define

$$\frac{\partial \vec{A}}{\partial x} = \lim_{h \to 0} \frac{\vec{A}(x+h, y, z) - \vec{A}(x, y, z)}{h}$$

$$\frac{\partial \vec{A}}{\partial y} = \lim_{k \to 0} \frac{\vec{A}(x, y+k, z) - \vec{A}(x, y, z)}{k}$$

$$\frac{\partial \vec{A}}{\partial z} = \lim_{l \to 0} \frac{\vec{A}(x, y, z+l) - \vec{A}(x, y, z)}{l}$$

These are the partial derivatives of $\vec{A}$ w.r.t. x, y, z

Then we can define higher order derivatives as

$$\frac{\partial^2 \vec{A}}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial \vec{A}}{\partial x}\right)$$

$$\frac{\partial^3 \vec{A}}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 \vec{A}}{\partial x^2}\right)$$

$$\frac{\partial^2 \vec{A}}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial \vec{A}}{\partial y}\right)$$

$$\frac{\partial^3 \vec{A}}{\partial y^3} = \frac{\partial}{\partial y}\left(\frac{\partial^2 \vec{A}}{\partial y}\right)$$

$$\vdots$$

& $$\frac{\partial^2 \vec{A}}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial \vec{A}}{\partial y}\right)$$

and so on.

If $\vec{A}$ has continuous partial derivatives then

$$\frac{\partial^2 \vec{A}}{\partial x \partial y} = \frac{\partial^2 \vec{A}}{\partial y \partial x}.$$

If $\vec{A} = (xy - x^2)i + (e^{xy} - y\cos x)j + yx\cos z\,\hat{k}$

Find $\frac{\partial \vec{A}}{\partial x}$, $\frac{\partial \vec{A}}{\partial y}$, $\frac{\partial \vec{A}}{\partial z}$

<u>Ans:</u>

$$\frac{\partial \vec{A}}{\partial x} = (y - 2x)\hat{i} + (ye^{xy} + y\sin x)\,j + y\cos z\,\hat{k}$$

$$\frac{\partial \vec{A}}{\partial y} = (x)\hat{i} + (xe^{xy} - \cos x)\hat{j} + x\cos z\,\hat{k}$$

$$\frac{\partial \vec{A}}{\partial z} = -\,yx\sin z\,\hat{k}$$

<u>Ex:</u> Also find $\frac{\partial^2 \vec{A}}{\partial x^2}$ & $\frac{\partial^2 \vec{A}}{\partial y^2}$

Verify that $\frac{\partial^2 \vec{A}}{\partial x \partial y} = \frac{\partial^2 \vec{A}}{\partial y \partial x}$.